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GENERALIZED THEORY FOR NONISOTHERMAL STRAIN

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The problem is solved for analytical description of relationships between strains, stresses, and temperature "at a point" of a solid during its thermomechanical loading. Stresses are divided into equilibrium and nonequilibrium. Equilibrium stresses do not depend on time effects for deformation, and through them by methods of classical plasticity theory irreversible strain is determined. Recovery of mechanical properties at elevated temperatures is considered. Apart from nonisothermal plastic strain, nonsteady and high-temperature steady-state creep and thermal aftereffect are described.

1. Basic Assumptions. Consideration is given to the condition of a constant density substance in the quite small neighborhood of a point of deforming solid as an element characterizing the condition at this point. Due to the specific nature of strains for a solid, this phenomenological element of material forms a closed thermodynamic system in which classical thermodynamic laws operate [1]. Experiments show [2, 3] that if at a certain instant of thermomechanical loading for an actual solid strain and temperature are fixed, then after this there is a reduction in stress so that complete (thermodynamic) equilibrium in the system sets in only after this relaxation.

Stresses which remain after transfer of the system from an actual to a thermodynamic equilibrium condition we call equilibrium, and the relaxed part of stresses whose tensor components are obtained by subtracting the equilibrium stress tensor component $\langle\sigma_{ij}\rangle$ from the corresponding stress tensor component σ_{ij} occurring at the instant of fixing strains and temperature, we call nonequilibrium stresses and we designate them ψ_{ij}. Thus, in each instant of deformation there is an identity

$$\psi_{ij} \equiv \sigma_{ij} - \langle\sigma_{ij}\rangle,\ i,\ j = 1, 2, 3. \tag{1.1}$$

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By definition nonequilibrium stresses are expressed directly in the form of thermodynamic nonequilibrium, i.e., deformation kinetics for an actual solid in time, displaying a zero beginning of thermodynamics for the case in question.

In thermodynamics the type of material in the sense of a "rheological rule" for the test material [1] depends entirely on the specific nature of its dissipation. With a known dissipation function relationships governing the link between stresses and strains at a point of the material, i.e., the type of material, may be found easily.

The total density for entropy production at the point of a material, i.e., the dissipative function of a phenomenological element within the bounds of nonequilibrium thermodynamics, is found from the first and second beginnings of thermodynamics in [4] in the form

$$\Theta = \left(\langle\sigma_{ij}\rangle \dot{\varepsilon}_{ij}^{H} + \psi_{ij}\dot{\mu}_{ij} - q_i T_{,i}/T\right)/T, \tag{1.2}$$

and according to the second law of thermodynamics, $\theta \geq 0$. Here and below $\varepsilon_{ij}{}^{H} = \varepsilon_{ij} - \varepsilon_{ij}{}^{r}$; ε_{ij}, $\varepsilon_{ij}{}^{H}$, $\varepsilon_{ij}{}^{r}$ are tensor components for total, irreversible, and reversible relative strains respectively; μ_{ij} are components of the tensor value characterizing nonequilibrium of the deformation process "by coordinates;" q_i are components of the velocity vector for heat flow; for repeated indices there is summing; a full stop above a symbol indicates differentiation with respect to time t, and a comma at the level of indices indicates differentiation with respect to the index following it.

2. Fundamental Equations. In future we limit ourselves to processes occurring in a field of uniform temperatures where the third term of (1.2) equals zero, and we analyze the remaining terms taking account of the fact that with the small strains in question their effect on each other may be ignored.

The first term of (1.2) $\langle\sigma_{ij}\rangle\dot{\varepsilon}_{ij}{}^{H} \geq 0$ indicates that in considering process kinetics irreversible strain is entirely determined only by equilibrium stresses. Irreversible deformation processes, composed of a sequence of equilibrium state, do not depend on time and they are a subject of classical plasticity theory [5]. In particular, with a smooth loading surface the associated flow rule is written as

$$d\varepsilon_{ij}^{H} = F \frac{\partial f}{\partial \langle s_{ij}\rangle} \frac{\partial f}{\partial \langle s_{mn}\rangle} d\langle s_{mn}\rangle, \quad \frac{\partial f}{\partial \langle s_{mn}\rangle} d\langle s_{mn}\rangle \geqslant 0. \tag{2.1}$$

For simplicity we limit ourselves to an initially isotropic material with translational strengthening, with which the surface loading equation has the form [6]

$$f(\langle\sigma_{ij}\rangle) = \frac{3}{2}\left(\langle s_{ij}\rangle - c\varepsilon_{ij}^{H}\right)\left(\langle s_{ij}\rangle - c\varepsilon_{ij}^{H}\right) - \langle\sigma\ \rangle^2 = 0, \tag{2.2}$$

where $\langle\sigma_y\rangle$ is yield strength with "equilibrium" loading; $\langle\sigma_{ij}\rangle$ are equilibrium stress tensor deviator components; $f = f(\langle\sigma_{ij}\rangle)$ is loading surface; F is a material constant.

According to the second term of (1.2) $\psi_{ij}\dot{\mu}_{ij} \geq 0$, assuming that $\psi_{ij} = \psi_{ij}(\mu_{11}, \mu_{12}, \mu_{13}, \ldots, \mu_{33})$ and that these functions are linear, we write

$$\psi_{ij} = r_{ijmn}\mu_{mn}. \tag{2.3}$$

As for every "coordinate" parameter of state, μ_{ij} are expressed in terms of independent "force" coordinates $\langle\sigma_{ij}\rangle$, ψ_{ij}, T, so that if for infinitely small time intervals dt equilibrium stresses have an increment $d\langle\sigma_{mn}\rangle$, nonequilibrium stresses have $d\psi_{mn}$, and temperature has dT, then

$$d\mu_{ij} = a_{ijmn} d\langle\sigma_{mn}\rangle + b_{ijmn} d\psi_{mn} + r_{ij} dT. \tag{2.4}$$

From (2.3) and (2.4) we obtain $\dot{\psi}_{ij} + A_{ijmn}\psi_{mn} = q_{ijmn}\langle\dot{\sigma}_{mn}\rangle + g_{ij}\dot{T}$ (A_{ijmn}, q_{ijmn}, g_{ij} are material constants).

For an initially isotropic material these relationships have the form

$$\dot{\psi}_{ij} + g\psi_{ij} = a\langle\dot{\sigma}_{ij}\rangle \tag{2.5}$$

Here coefficients g and *a* only depend on temperature. Equations (1.1), (2.1), and (2.5) determine irreversible strain "at a point" of an initially isotropic solid.

3. Simple Uniaxial Tension and Nonsteady-State Creep with a Constant Holding Temperature. Separation of total stresses into equilibrium and nonequilibrium makes it possible to describe a set of strain rate effects: the dependence of strain on loading rate, a delay in creep, a dependence of the additional loading on loading schedule, nonsteady-state creep, which have been studied in [7] on the basis of slip concepts.

As an example we write analytically simple uniaxial tension and nonsteady-state creep at constant room holding temperature.

With uniaxial loading from (2.2)

$$\langle\sigma\rangle = \langle\sigma_y\rangle + \frac{3}{2}c\varepsilon^{H}, \tag{3.1}$$

and from (2.5)

$$\dot{\psi} + g\psi = a\langle\dot{\sigma}\rangle \tag{3.2}$$

(g and a are material constants).

By substituting (1.1) and (3.1) in (3.2), and solving the equation obtained with initial conditions (t = 0, $\sigma(0) = \sigma_y$, $\langle\sigma\rangle(0) = \langle\sigma_y\rangle$, $\varepsilon^H(0) = 0$), we find the expression

$$\begin{aligned}\varepsilon^{H}(t) = \frac{2}{3c(1+a)}\Bigg[\sigma(t) + \frac{ag}{1+a}\int\limits_0^t \sigma(\tau)\exp\left(-\frac{g}{1+a}(t-\tau)\right)d\tau + \\ + ((1+a)\langle\sigma_y\rangle - \sigma_y)\exp\left(-\frac{g}{1+a}t\right) - (1+a)\langle\sigma_y\rangle\Bigg],\end{aligned} \tag{3.3}$$

determining irreversible strain in relation to extension rate.

Now we establish the link between σ_y and $\langle\sigma_y\rangle$. By substituting (1.1) in (3.2) and solving the equation obtained relative to $\langle\sigma\rangle$ with extension to the yield limit, i.e., with t = 0, $\sigma(0) = \langle\sigma\rangle(0) = 0$, we have

$$\langle\sigma\rangle(t) = \frac{1}{1+a}\left[\sigma(t) + \frac{ag}{1+a}\int\limits_0^t \exp\left(-\frac{g}{1+a}(t-\tau)\right)\sigma(\tau)\,d\tau\right]. \tag{3.4}$$

As a rule, extension in the elastic region is carried out by a linear rule $\sigma(t) = vt$ (v = const). By substituting this value in (3.4) and considering that at the yield limit $\sigma_y = vt_y$, we write

$$\langle\sigma_y\rangle = \sigma_y - \frac{av}{g}\left[1 - \exp\left(-\frac{g}{1+a}\frac{\sigma_y}{v}\right)\right]. \tag{3.5}$$

The "equilibrium" yield limit $\langle\sigma_y\rangle$ is a material constant. In practice it is the yield limit obtained by experiment whose value does not change with quite low extension rates. Therefore, according to (3.5) the total yield limit $\sigma_y = f(v)$, i.e., Eq. (3.5) makes it possible to determine σ_y as a function of extension rate. However, the practical value of Eq. (3.5) consists of the fact that it makes it possible by means of very simple tensile tests up to σ_y with different rates to find constants g and a. For this purpose it is sufficient to carry out two experiments in tension: one for time t_1 with constant rate v_1 to $\sigma_1{}^y$, and the second, for example for time $t_2 = 2t_1$ with constant rate v_2 to $\sigma_2{}^y \neq \sigma_1{}^y$. By substituting these values in (3.5) and solving the equations obtained, we find that

$$\frac{a}{g} = \frac{t_1 - \langle\sigma_T\rangle/v_1}{1 - \exp(\ln y)}, \quad \frac{g}{1+a} = -\frac{1}{t_1}\ln y, \tag{3.6}$$

where

$$y = \frac{\left(2t_1 - \frac{\langle\sigma y\rangle}{v_2}\right) \pm \left[\left(2t_1 - \frac{\langle\sigma y\rangle}{v_2}\right)^2 - 4\left(t_1 - \langle\sigma_y\rangle\frac{v_2 - v_1}{v_1 v_2}\right)\left(t_1 - \frac{\langle\sigma y\rangle}{v_1}\right)\right]^{1/2}}{2\left(t_1 - \frac{\langle\sigma y\rangle}{v_1}\right)}.$$

Placing stresses of constant value $\sigma = \sigma_* =$ const in Eq. (3.3), we have

$$\varepsilon^{\mathrm{H}}(t)=\frac{2}{3c(1+a)}\left[\sigma_*-\sigma_y\exp\left(-\frac{g}{1+a}t\right)+(\sigma_*a-(1+a)\langle\sigma_y\rangle)\quad\left(1-\exp\left(-\frac{g}{1+a}t\right)\right)\right]. \tag{3.7}$$

Subtracting from (3.7) the value of irreversible strain with instantaneous tension up to σ_*, we obtain an equation for irreversible strain with constant stress $\Delta\varepsilon_n^{\mathrm{H}}(t)=\frac{2}{3c(1+a)}(\sigma_*a-\sigma_y-(1+a)\langle\sigma_y\rangle)\left(1-\exp\left(-\frac{g}{1+a}t\right)\right)$, which determines nonsteady state creep after instantaneous tension.

4. <u>High-Temperature Strain</u>. Irreversible strain distorts the material structure. However, at sufficiently high temperature a structure distorted by strain recovers to the original natural condition. There is thermal loss of strength or material "recovery." This loss of strength is a specific feature of high-temperature strain generating qualitatively new effects (steady-state creep and thermal aftereffect [8, 9]).

Now we limit ourselves to material with linear translational strengthening with which the loading surface moves in a stress field as a rigid whole. Thermal loss of strength may be expressed by movement of the center of the loading surface to the initial position which it occupied before directional strain hardening. As a thermally activated process this loss of strength proceeds with time, and in order to take account of this the associated flow rule (2.1) is written in the form

$$d\varepsilon_{ij}^{\mathrm{H}}=F\frac{\partial f}{\partial\langle s_{ij}\rangle}\frac{\partial f}{\partial\langle s_{mn}\rangle}d\Sigma_{mn}; \tag{4.1}$$

$$d\Sigma_{mn}=d\langle s_{mn}\rangle+K(T)\alpha_{mn}dt, \tag{4.2}$$

and the loading surface equation is written in the form

$$f(\langle\sigma_{ij}\rangle)=\frac{3}{2}(\langle s_{ij}\rangle-\alpha_{ij})(\langle s_{ij}\rangle-\alpha_{ij})-\langle\sigma_y\rangle^2=0; \tag{4.3}$$

$$d\alpha_{ij}=cd\varepsilon_{ij}^{\mathrm{H}}-K(T)\alpha_{ij}dt. \tag{4.4}$$

The first term of (4.4) indicates that material strengthening is caused by irreversible strain. In particular, with K(T) = 0 and zero initial conditions $\alpha_{ij}=c\varepsilon_{ij}^{H}$, i.e., with equlibrium processes, a case of plasticity theory occurs [6]. The second term of (4.4) reflects the property of the material to weaken at quite high temperatures, and therefore parameter K depends on temperature so that in the absence of this weakening K(T) = 0.

Equations (1.1), (2.5), (4.1)-(4.4) are a complete system of rheological equations for nonisothermal strain taking account of thermal loss of strength. We use them for analytical description and explanation of conditions for the existence of such specific high-temperature strain effects as steady-state creep and thermal aftereffect.

5. <u>Steady-State Creep</u>. We assume that steady-state creep proceeds at the same constant temperature T_* as prior loading. This temperature is sufficiently high so that equilibrium stresses do not generally exist and there is high-temperature loss of strength so that at any arbitrary instant of time $\psi_{ij}=0$, and consequently $\langle\sigma_{ij}\rangle=\sigma_{ij}$. In this way in (3.2) $a=0$ and the relationship between stresses and irreversible strains with instantaneous prior loading to $\sigma_{ij}=\sigma_{ij}^*$, $\sigma_{ij}^*=\mathrm{const}$ is represented by relationships (4.1)-(4.4) with dt = 0, i.e., equations of a flow theory variant [6].

The strain mechanism under conditions of thermal loss of strength with constant stresses and temperatures is explained on the example of the behavior of the specimen after simple uniaxial tension when in accordance with (4.4)

$$d\alpha=cd\varepsilon^{\mathrm{H}}-K(T)\alpha dt. \tag{5.1}$$

Let a specimen of initially isotropic material under thermal loss of strength conditions be extended at constant temperature to $\sigma_*>\sigma_y$, and then stress be kept constant. With $\sigma_*=\sigma_*>\sigma_y$, there is thermal loss of strength, as a result of which the center of the loading surface O_1', occupied at the instant of reaching position A with stress value σ_* (Fig. 1, where the initial position of the yielding sphere is indicated by broken lines), returns to the original position O, and for time intervals dt it is displaced by elementary distance $O_1'O_1$, so that the loading surface itself occupies position B. In this way irreversible strain does not occur ($d\varepsilon^H=0$) and from (5.1)

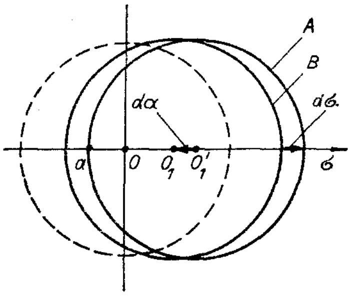

Fig. 1

$$d\alpha = -K(T)\alpha dt. \tag{5.2}$$

The change in position of a rigid loading surface (sphere of constant radius σ_y), caused by elementary displacement of its center by a value dα determined by Eq. (5.2) with a fixed tensile stress, means its increase in relation to the new position of this surface by an elementary value dσ = dα (Fig. 1). As is well known from flow theory, this causes an instantaneous elementary increase in irreversible strain $d\varepsilon^{H}$, which strengthens the material, returning the loading surface to the original condition (from position B to position A).

With an instantaneous elementary increment in stress the second term in (4.4) is absent, and consequently

$$d\sigma = |d\alpha| = cd\varepsilon^{H}. \tag{5.3}$$

Since the loading surface shifts as a rigid whole and dσ = dα, then from (5.2) and (5.3)

$$\dot{\varepsilon}^{H}(t) = K(T_*)\,\alpha(t)/c. \tag{5.4}$$

On the other hand, by substituting (4.3) in (4.1) for uniaxial loading we obtain taking account of (4.2) and (4.4) with dν = 0

$$\dot{\varepsilon}^{H}(t) = 4F\sigma_y^2 K(T_*)\,\alpha(t). \tag{5.5}$$

By comparing (5.4) and (5.5) we find material constant $c = 1/(4F\sigma_y^2)$.

With a constant loading condition when according to (4.3) α_{ij} (and this also means α with uniaxial loading) is a fixed value, Eq. (5.4) determines irreversible strain with a constant rate.

Thus, simultaneous occurrence of mechanical strengthening and mechanical recovery processes with a constant load condition leads to irreversible strain with a constant rate, i.e., to steady-state (linear) creep.

It is easy to see that formal substitution in (5.1) of the value α = const leads to a similar result [i.e., to Eq. (5.4)] since the reasoning given above is only necessary for explaining the steady-state creep mechanism. A change in loading surface due to recovery of mechanical properties, but not connected with creep strain, was noted in [10].

6. Thermal Aftereffect. The phenomenon of a "spontaneous" change in article shape with repeated cyclic changes in temperature, which may also occur in the absence of an external load, is called the thermal aftereffect. With a change in temperature for a polycrystalline body, due to the dense fitting in it of randomly placed particles with each other (crystals, grains, etc.) thermal microstructural stresses arise (second order) [7, 11]. Equality to zero of the average microstructural stresses for a phenomenological element free from external forces of a body with any gradient-free change in its temperature points to their probable random distribution. Therefore, with a change in temperature they increase identically in all directions, reaching first the flow surface or all of its points at once (if this surface is a sphere with a center at the zero point of the stress field), or at a point of it which is least distant from the stress field (see Fig. 1, point a). With a prolonged increase in thermostructural stresses the body macrodeforms irreversibly in a direction determined by the position of the "minimum" point on the loading surface (thermal reduction in length in Fig. 1).

If a strengthening material deforms irreversibly in a certain direction directly due to a change in temperature, then in this direction the loading surface is removed from the zero center of the stress field, i.e., there is thermal strengthening. Therefore, if the temperature returns to the original value, and then it changes again as previously, then for the start of irreversible strain in the previous direction it should reach the maximum previous value, and a change in the shape of the body will not occur with cyclic single-value changes in temperature. However, this claim is only true in the absence of thermal loss of strength. In the presence of it with a fixed change in temperature the material weakens. This weakening will be accompanied by irreversible strain whose mechanism is the same as for steady-state creep, only here it forms with a relaxation reduction of thermostructural microstresses which make its rate variable (dying down gradually).

A predominant direction for irreversible strain in a material due to a change in temperature may also be created by applied external forces. In fact, if some external load is applied to a body not taking the material out of the region of elastic strains, then with a change in temperature thermostructural stresses will combine with force stresses in a certain direction. If the change in temperature is sufficiently large that this total value of stresses exceeds the yield limit, then in this direction with a change in temperature irreversible strain occurs at once. In addition, if there is thermal loss of strength, then the shape of the article changes with cyclic single-amplitude thermal changes in a similar way to that in question.

Thermostructural stresses occurring in an article with a change in its temperature are characterized by a certain average isotropic measure which is introduced into the associated flow rule. This measure, reflecting the properties of these stresses, should depend on the whole history of temperature changes: decrease with the passage of time at constant temperature and give an increment in irreversible strain both with an increase and a decrease in temperature. These requirements are satisfied by a scalar value

$$Q(t) = \left|\int_0^t \dot{T}(s)\, R(t-s)\, ds\right|, \tag{6.1}$$

where $R(t - s)$ is a decreasing function with an increase in argument $t - s$, describing relaxation of thermostructural microstresses. Therefore, nucleus R in (6.1) is taken in the form of (6.2) $R = B\exp(-\beta(t - s))$ (B is a constant, and β is a material parameter).

A measure of thermostructural microstresses is added to existing stresses under conditions of thermal loss of strength by writing (4.2) in the form

$$d\Sigma_{mn} = d\langle s_{mn}\rangle + K(T)\alpha_{mn}dt + dQ(t). \tag{6.3}$$

First we find irreversible strain due to an instantaneous change in temperature $\Delta T = T_{*} - T_0$ (i.e., with thermal shock) with which $dt = 0$, and consequently, $d\langle s_{mn}\rangle = 0$. Since in the case of simple uniaxial loading with tension by substituting (4.3) in (4.1) we have

$$d\varepsilon^{H} = 4F\langle\sigma_{T}\rangle^{2}d\Sigma, \tag{6.4}$$

and according to (6.3) $d\Sigma = Bd\left[\int_0^t \dot{T}(s)\exp(-\beta(t-s))\,ds\right]$, and by integrating (6.4) and considering that with $t^0 = 0$ $\Delta T = 0$, $\varepsilon^H = \varepsilon^H(0) = \varepsilon_0{}^H$ is irreversible strain at the start of thermal shock, $F = 1/(4c\sigma_y{}^2)$, we obtain

$$\varepsilon_{*}^{H} = \varepsilon_0^{H} + B|\Delta T|/c \tag{6.5}$$

($\varepsilon_{*}{}^H$ is irreversible strain at the end of thermal shock).

Now we find irreversible strain as a result of thermal shock. For this purpose we fix changed temperature and tensile stress, i.e., we assume that $T = T_{*} = \text{const}$ and $\sigma = \sigma_{*} = \text{const}$, and by solving set of Eqs. (5.1) and (6.3) we have

$$c\varepsilon^{H} = c\Delta\varepsilon_{*}^{H} + K\alpha_{*}t + \frac{B}{\beta}\left[Kt - \left(1-\frac{K}{\beta}\right)\left(1-\exp(-\beta t)\right)\right]|\Delta T|. \tag{6.6}$$

For uniaxial tension from (4.3) we find the equality $(3/2)\alpha = \langle\sigma\rangle - \langle\sigma_y\rangle$, which with high-temperature creep without strengthening, i.e., with $\langle\sigma\rangle = \sigma = \sigma_{*} = \text{const}$ and $\alpha = \alpha_{*} = \text{const}$, takes the form $\alpha_{*} = (2/3)(\sigma_{*} - \sigma_y)$. By substituting this value of α_{*} and $\Delta\varepsilon_{*}{}^H = \varepsilon_{*}{}^H - \varepsilon_0{}^H$ and from (6.3) in (6.6) we obtain the main calculation equation

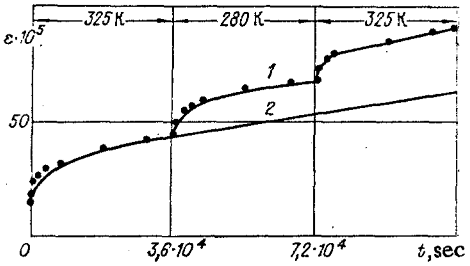

Fig. 2

$$\varepsilon^{\text{н}}(t)=\frac{1}{c}\left[\frac{2c}{3}K(\sigma_*-\sigma_y)t+B\left(1-\frac{K}{\beta}t-\frac{1}{\beta}\left(1-\frac{K}{\beta}\right)(1-\exp(-\beta t))\right)|\Delta T|\right], \tag{6.7}$$

which takes account of the main characteristic features of strain with temperature oscillation under conditions of high-temperature creep noted in experiments [8, 9, 11].

7. Creep of Zinc with Cyclic Thermal Changes. In Fig. 2, curve 2 is creep strain for polycrystalline zinc with T_1 = 325 K and σ_* = 60 MPa, and shown here also curve 1 is its strain with cyclically changing temperature [8]. From the diagram it follows that cyclic changes in temperature repeatedly increase the creep of zinc making irrefutably important to consider these features in theory. The calculation equation for strain under the conditions described emerges directly from Eq. (6.7)

$$\Delta\varepsilon^{\text{н}}(t)=[L+Rt+M(1-\exp(-\beta t))]|\Delta T|, \tag{7.1}$$

where L, R, M, and β are material constants subject to determination by experiment.

We find constant L, characterizing the instantaneous increase in strain with a jump in temperature, by using the test results for zinc given in [8, 9, 12], from which it follows that with ΔT = 40 K and σ_* = 60 MPa with an increase in temperature $\Delta\varepsilon^H = 7.7\cdot10^{-5}$. By substituting these values in (7.1) with t = 0 we obtain $L_B = 1.925\cdot10^{-6}$ K^{-1}. With a reduction in temperature $\Delta\varepsilon^H = 5.55\cdot10^{-5}$ and $L_H = 1.387\cdot10^{-6}$ K^{-1}. Here L_B and L_H are values of L with increased and reduced temperature.

In order to determine constant M, characterizing the thermal aftereffect, again we use results given in [8, 9, 12] according to which with ΔT = 40 K and σ_* = 60 MPa the thermal aftereffect for zinc forms strain $\varepsilon_1{}^H = 3.3\cdot10^{-5}$. By substituting these data in (7.1) with L = R = 0 and $t\to\infty$, we have $M = 8.24\cdot10^{-7}$ K^{-1}.

Analysis of strain curves for the thermal aftereffect given in [8, 9, 12] indicate that in zinc it ceases not later than 2 h after the temperature jump. By taking relaxation time t_p = 7200 sec, we obtain from (7.1) (with L = R = 0) $\beta = 5\cdot10^{-3}$ $\sec^{-1}$.

In order to determine constant R we use the isothermal creep curve 2 (Fig. 2) according to which after 10 h with T = 325°K, when all of the relaxation processes caused by a change in temperature have occurred, $\varepsilon^H = 4.25\cdot10^{-4}$. In this way from (7.1) we find $R = 1.86\cdot10^{-10}$ $(K\cdot\sec)^{-1}$.

By substituting values of constants found in (7.1) we determine the calculation equations sought for strain in zinc with cyclic thermal changes. Results of calculations corresponding to test data are shown in Fig. 2.

The theory obtained for nonisothermal strain expresses a new approach to the phenomenon of solid deformation confirmed for some more typical cases of loading by test data.

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NONDISSIPATIVE INELASTIC STRAIN FOR A SOLID ELEMENT

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Elastic strains for a solid element are the part of strain for the element which disappears after it is unloaded (removal of external effects). Inelastic (residual) strains are the part of strain for the element which remains in it after it is unloaded. Apart from inelastic strains for an element with which mechanical energy is converted into heat, nondissipative inelastic strains are possible, i.e., those with which mechanical energy is not converted into heat.

One of the simplest and graphic examples of a deformation process with nondissipative inelastic strains may be deformation of a system of two elastic springs and a rod (Fig. 1 from [1]). During deformation of this system mechanical energy is not converted into heat, but due to overall clamps A the unloading process proceeds in a different way from the loading process, as a result of which the relationship between force p applied to the rod and displacement Δ of the rod will have the form indicated in Fig. 2, where Δ^* is nondissipative inelastic strain of the system.

In this work equations are formulated determining nondissipative inelastic strains for a solid element.

1. Division of Strain into Elastic and Inelastic. As a strain tensor we take [2]

$$\varepsilon_{ij}e^{i}e^{j}=\widehat{\varepsilon}_{\alpha\beta}\text{Э}^{\alpha}\text{Э}^{\beta},\quad \widehat{\varepsilon}_{\alpha\beta}=\frac{1}{2}(\text{Э}_{\alpha}\cdot\text{Э}_{\beta}-\mathbf{e}_{\alpha}\cdot\mathbf{e}_{\beta}) \qquad (1.1)$$

(Э^{α} and e^{i} are basis vectors of Lagrangian and Cartesian coordinate systems).

The state of a material element from which strain is reckoned is called the initial state. We assume that stresses in the initial state equal zero, density and temperature equal prescribed values ρ_0 and T_0 differing from zero, and in any stage of the deformation process for the material element it is possible to "unload it completely" to a state with stress and temperature T_0 equal to zero (by cutting an element from a material, heating and cooling it to the temperature of the initial condition, and giving it the possibility of deforming freely).

In gaseous media equality of stresses to zero is only possible with density equal to zero. In this case as an initial condition we take that in which the average stress and temperature equal a prescribed value differing from zero, and by "complete unloading" we

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